Exercise 39

For the following exercises, solve the equations over the complex numbers.

$$2x^2 + 2x + 5 = 0$$

Solution

Factor the coefficient of x^2.

$$2\left(x^2 + x + \frac{5}{2}\right) = 0$$

The two terms with x, x^2 and x, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = 1$, which means $B = \frac{1}{2}$ and $B^2 = \frac{1}{4}$. Add and subtract $\frac{1}{4}$ within the parentheses on the left side and apply the identity.

$$2\left[\left(x^2 + x + \frac{1}{4}\right) + \frac{5}{2} - \frac{1}{4}\right] = 0$$

$$2\left[\left(x + \frac{1}{2}\right)^2 + \frac{9}{4}\right] = 0$$

$$2\left(x + \frac{1}{2}\right)^2 + \frac{9}{2} = 0$$

Now that x appears in only one place, it can be solved for. Subtract 9/2 from both sides.

$$2\left(x + \frac{1}{2}\right)^2 = -\frac{9}{2}$$

Divide both sides by 2.

$$\left(x + \frac{1}{2}\right)^2 = -\frac{9}{4}$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{\left(x + \frac{1}{2}\right)^2} &= \sqrt{-\frac{9}{4}} \\ &= \sqrt{\frac{9}{4}(-1)} \\ &= \sqrt{\frac{9}{4}}\sqrt{-1} \\ &= \frac{3}{2}i\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x + \frac{1}{2}$.

$$\left|x + \frac{1}{2}\right| = \frac{3}{2}i$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x + \frac{1}{2} = \pm\frac{3}{2}i$$

Subtract 1/2 from both sides.

$$x = -\frac{1}{2} \pm \frac{3}{2}i$$

Therefore,

$$x = \left\{-\frac{1}{2} - \frac{3}{2}i, -\frac{1}{2} + \frac{3}{2}i\right\}.$$